

# Modeling and Control Design of Quad copter Failsafe System

Chun-Yi Lin and Wu-Sung Yao

Department of Mechanical and Automation Engineering,  
National Kaohsiung First University of Science and Technology,

**Abstract**—Quadcopter Unmanned Aerial Vehicle (UAV) has adapt to rapid response and high terrain capability, and now widely be used in reconnaissance, transporting supplies etc. In quad copter control system, with four degrees of freedom, quad copters are difficult to be controlled by non-linear system under vibration from structure failure. In this paper, a dynamics model of quad copter control functionis presented. Radial Basis Function Neural Network (RBFNN) based on Proportional-Integral-Derivative (PID) controller is designed to realize the Adapted and automatically manage Control System (ASAC).Experiment of a quad copter in flight is given to verify the proposed scheme.

**Index Terms**—Quadcopter, Unmanned Aerial Vehicle, Radial Basis Function Neural Network

## I. INTRODUCTION

A flight control UAV system provides enabling technology for the aerial vehicles to fulfill the flight missions, especially when the missions are often planned to perform tedious tasks under extreme flight cases which are not suitable for piloted operation. Most UAVs nowadays are used for surveillance and reconnaissance purposes, and few cases for heavy payload delivery. It well known that UAV flight control systems are often considered safety/mission critical, as a flight failure easy result in loss of the UAV. In these cases a significant portion of the UAVs in operation remain remotely piloted, with autonomous flight control restricted to attitude hold, non-agile way-point flight, or loiter maneuvers. Linear controllers are typically adequate for these maneuvers. In general, UAVs are often used to perform agile maneuvers in the presence of significant model and environmental uncertainties. Due to UAV dynamics being inherently nonlinear, therefore, linear control method can only be guaranteed to be locally stable and difficult to extract the desired performance with the presence of significant nonlinear effects. Nonlinear control techniques should be utilized to consider for wrong linear models and nonlinear kinematic effects [1][2]. The tuning process, whereby the optimum values for the controller parameters are obtained, is a critical challenge. Many studies were conducted to find a way for tuning controller parameters in order to get adequate performances such as fast response, zero steady-state error, and minimum overshoot/undershoot. Although the traditional PID control system cannot be used to solve the problem of structural parameters without an accurate mathematical model. PID control is most wide used in the flight control system for cheap, simple design, and easy to setup. For nonlinear system, the identified error of the system

parameters easily cause instable just only using trial and error method. For structural damage or parts failure, the controller also must be made to adapt to the varying parameters of the controlled plant. While the control parameters must be adjusted manually within a limited range. The accident to getting out of control is often in short period of time (about 0.1 seconds). In the field of mathematical modeling, a radial basis function network is an artificial neural network that uses radial basis functions as activation functions. The output of the network is a linear combination of radial basis functions of the inputs and neuron parameters [2][3]. Radial basis function networks have many been used, including of function approximation, time series prediction, classification, and system control. Therefore. PID control with RBFNN are given in this paper, including the control variables following the command of quadcopter flight control system response and interference. Using a quadcopter UAV proposed based on advanced PID control for microcontroller is analyzed with implementation.

## II. ANALYSIS OF QUADCOPTER MATHEMATICAL MODEL

In this section, the mathematical model of the quadcopter is addressed, where the definitions of the notations are given in Table 1.

**Table 1. Definitions of the notations**

$\omega_1$	Angular Velocity of rotor 1
$\omega_2$	Angular Velocity of rotor 2
$\omega_3$	Angular Velocity of rotor 3
$\omega_4$	Angular Velocity of rotor 4
$T_1$	Torque of rotor 1
$T_2$	Torque of rotor 2
$T_3$	Torque of rotor 3
$T_4$	Torque of rotor 4
$e_x$	Displacement of x axis
$e_y$	Displacement of y axis
$e_z$	Displacement of z axis
$f_1$	Throttle Force of rotor 1
$f_2$	Throttle Force of rotor 2
$f_3$	Throttle Force of rotor 3
$f_4$	Throttle Force of rotor 4
$mg$	Weight of whole quadcopter
$\psi$	Yaw axis angular displacement
$\theta$	Pitch axis angular displacement
$\Phi$	Roll axis angular displacement

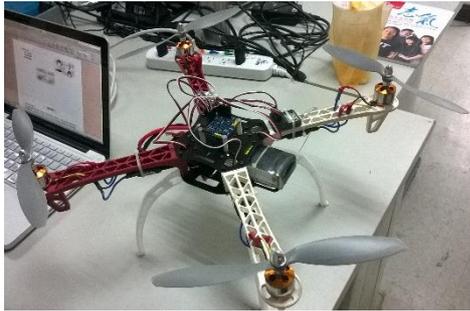


Fig. 1 Proposed control system of the quad copter

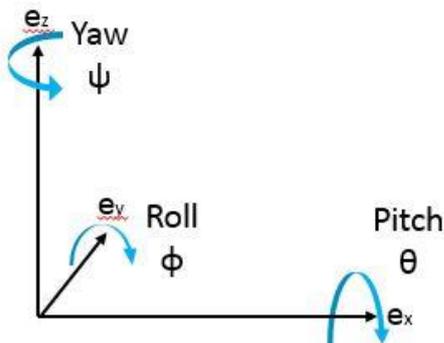


Fig.2 Coordinate of Roll ( $\phi$ ), Pitch ( $\theta$ ), Yaw ( $\psi$ )

As shown in Fig. 2, the inertial frame of quadcopter can be defined as  $\mathbf{x} = (x, y, z)$  and  $\dot{\mathbf{x}} = (\dot{x}, \dot{y}, \dot{z})$ , [4]. Similarly, Roll, Pitch and Yaw can be defined as  $\boldsymbol{\theta} = (\phi, \theta, \psi)$  in the body frame. The corresponding angular velocities  $\dot{\boldsymbol{\theta}} = (\dot{\phi}, \dot{\theta}, \dot{\psi})$  can be obtained, where  $\boldsymbol{\omega}$  should be noted in this paper.

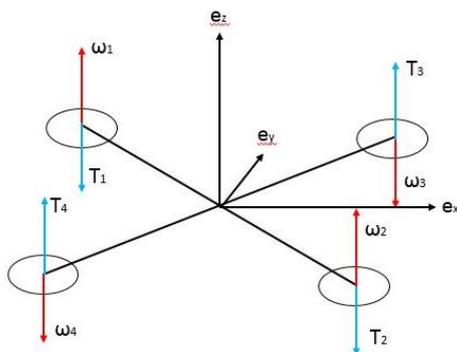


Fig.3 Angular velocity vector and torque of each axis

As shown in Fig.3, the angular velocity is defined along the axis of this rotation. And  $\dot{\boldsymbol{\theta}}$  is the time derivative of yaw, pitch, and roll, where we can obtain the equation as (1).

$$\boldsymbol{\omega} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_\phi & c_\theta \\ 0 & -s_\phi & c_\theta \end{bmatrix} \dot{\boldsymbol{\theta}} \quad (1)$$

where  $\boldsymbol{\omega}$  was the main angular velocity vector in the body frame. A rotation matrix  $\mathbf{R}$  between the body frame and inertial frame is given, where this matrix can be derived by using ZYZ Euler angle conventions and undoing the yaw, pitch, and roll as (2),

$$\mathbf{R} = \begin{bmatrix} c_\phi c_\psi - c_\theta s_\phi s_\psi & -c_\psi s_\phi - c_\phi c_\theta s_\psi & -s_\psi \\ c_\theta c_\psi s_\phi + c_\phi s_\psi & c_\phi c_\psi c_\theta - s_\phi s_\psi & c_\psi \\ s_\phi s_\psi & c_\phi s_\psi & c_\theta \end{bmatrix} \quad (2)$$

In order to building kinematics model, its physical properties

of the quadcopter should be previewed. The whole axis motion of vehicle is forced by motor as shown in Fig.4. Therefore, we can find the throttle of propeller by setting motor rotation speed. The relatively side of the propeller turning is inversed each other. If the left front and right behind propeller is the same rotation clockwise, then the right front and left behind should be counterclockwise. Therefore, the generated torques are balanced if all propellers are spinning at the same rate. In the inertial frame, the acceleration of the quadcopter is generated by thrust, gravity.

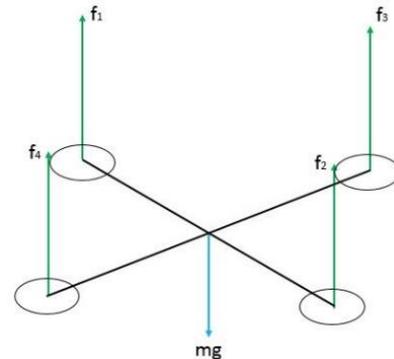


Fig.4 Throttle force and the gravity

We can obtain the thrust vector in the inertial frame by using our rotation matrix  $\mathbf{R}$  to mapping the thrust vector from the body frame to the inertial frame. Thus, this linear motion can be summarized as (3).

$$m\ddot{\mathbf{x}} = \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + \mathbf{R}\mathbf{T}_B + \mathbf{F}_D \quad (3)$$

where  $\mathbf{x}$  is the position of the quadcopter,  $g$  is acceleration of gravity,  $\mathbf{F}_D$  is the drag force vector and the thrust vector is  $\mathbf{T}_B$  in the body frame. The rotational equations of motion can be derived from Euler's equations for rigid body dynamics. Expressed in vector form Euler's equations are written as

$$\mathbf{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{I}\boldsymbol{\omega} = \boldsymbol{\tau}$$

where  $\boldsymbol{\omega}$  is the angular velocity vector,  $\mathbf{I}$  is the inertia matrix, and  $\boldsymbol{\tau}$  vector of external torques. Then, we can have the equation (4).

$$\dot{\boldsymbol{\omega}} = \begin{bmatrix} \dot{\omega}_x \\ \dot{\omega}_y \\ \dot{\omega}_z \end{bmatrix} = \mathbf{I}^{-1}(\boldsymbol{\tau} - \boldsymbol{\omega} \times (\mathbf{I}\boldsymbol{\omega})) \quad (4)$$

We can model the quadcopter as two thin uniform rods crossed at the origin with a point mass. It is clear that the symmetries result in a diagonal inertia matrix form, as the equation (5).

$$\mathbf{I} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \quad (5)$$

Thus, we can obtain mathematical model of (6).

$$\dot{\boldsymbol{\omega}} = \begin{bmatrix} \tau_\phi I_{xx}^{-1} \\ \tau_\theta I_{yy}^{-1} \\ \tau_\psi I_{zz}^{-1} \end{bmatrix} - \begin{bmatrix} I_{yy}^{-1} I_{zz}^{-1} \omega_y \omega_z \\ I_{xx}^{-1} \omega_x \omega_z \\ I_{zz}^{-1} I_{xx}^{-1} \omega_x \omega_y \\ I_{xx}^{-1} I_{yy}^{-1} \omega_x \omega_y \\ I_{zz}^{-1} \omega_z \omega_y \end{bmatrix} \quad (6)$$

System inputs are including of velocity vector, angular velocity vector, acceleration vector, and angular acceleration vector. Thus, we can control only motor throttle. Let  $\mathbf{x}_1$  is the

position of quadcopter,  $x_2$  is the linear velocity of quadcopter,  $x_3$  is angular velocity vector of Roll, Pitch and Yaw and  $x_4$  is angular velocity vector. Therefore, the state space equations can be given as shown in (7), (8), (9), and (10).

$$x'_1 = x_2 \tag{7}$$

$$x'_2 = \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} + \frac{1}{m} RT_B + \frac{1}{m} F_D \tag{8}$$

$$x'_3 = \begin{bmatrix} 1 & 0 & -s_\theta \\ 0 & c_\phi & c_\theta s_\phi \\ 0 & -s_\phi & c_\theta s_\phi \end{bmatrix}^{-1} x_4 \tag{9}$$

$$x'_4 = \begin{bmatrix} \tau_\phi I_{xx} & -1 \\ \tau_\theta I_{yy} & -1 \\ \tau_\psi I_{zz} & -1 \end{bmatrix} - \begin{bmatrix} I_{yy} - I_{zz} & \omega_y \omega_z \\ I_{zz} - I_{xx} & \omega_z \omega_x \\ I_{xx} - I_{yy} & \omega_x \omega_y \end{bmatrix} \tag{10}$$

With single gyro, only the angle derivatives  $\dot{\phi}$ ,  $\dot{\theta}$  and  $\dot{\psi}$  can be used. We would like to stabilize the quadcopter in a position at horizontal, so desired velocities and angles are zero with output torque  $\tau = Iu(t)$ , as shown in (11).

$$\begin{bmatrix} \tau_\phi \\ \tau_\theta \\ \tau_\psi \end{bmatrix} = \begin{bmatrix} -I_{xx} (K_d \dot{\phi} + K_p \int_0^T \dot{\phi} dt) \\ -I_{yy} (K_d \dot{\theta} + K_p \int_0^T \dot{\theta} dt) \\ -I_{zz} (K_d \dot{\psi} + K_p \int_0^T \dot{\psi} dt) \end{bmatrix} \tag{11}$$

### III. ANALYSIS OF RBFNN-PID CONTROL

Theoretically, more than three layers Back Propagation Neural Networks systems (BPNN)[3][5] can be used to approximate nonlinear function. Since BPNN at each input sample data, the Internet must re-adjust weights of all the links in the learning process. So learning speed cannot meet the immediate requirements for real time control system. Unlike BPNN, RBFNN is three layers forward network. Although the network of mapping input being non-linear, the hidden layer to the output layer mapping is linear[3]. The main difference that is the hidden layer in BPNN are using a sigmoid function, as the following formula (12).

$$f(x) = \frac{1}{1+e^{-x}} \tag{12}$$

The space of input value of this function is non-zero within a range of infinity. But the action function in RBFNN is Gaussian basis function as shown in (13).

$$f(x) = e^{-\|x - c_j\|^2} \tag{13}$$

The space of input value of this function is non-zero within a limited range. So RBFNN is a local approximation of neural network, which can be used to increase the learning speed [1][3]. RBFNN used in the control system can effectively improve the accuracy, robustness, and self-adaptive systems. In this study, PID control and RBFNN are integrated to design stable control system of the quadcopter, as shown in Fig. 5, where we have basic PID control equations as (14), (15), (16), and (17).

$$u(t) = k_p (e(t) + \frac{1}{T_i} \int_0^t e(t) dt + T_D \dot{e}(t)) \tag{14}$$

$$e_\theta = K_d \dot{\theta} + K_p \int_0^T \dot{\theta} dt + K_i \iint_0^T \dot{\theta} \tag{15}$$

$$e_\phi = K_d \dot{\phi} + K_p \int_0^T \dot{\phi} dt + K_i \iint_0^T \dot{\phi} dt dt \tag{16}$$

$$e_\psi = K_d \dot{\psi} + K_p \int_0^T \dot{\psi} dt + K_i \iint_0^T \dot{\psi} \tag{17}$$

Via intelligent algorithms, the parameters of PID can be adjusted without the accuracy mathematical model. An incremental algorithm of PID controller is given, where the error can be shown in (18).

$$e(k) = r_{in}(k) - y_o(k) \tag{18}$$

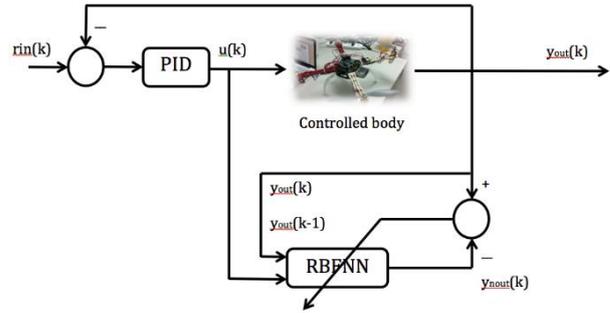


Fig. 5 The PID controller base on RBFNN

In Fig. 5, output of the PID controller value in (19) and which distinguish between the value forward as (20).

$$u(k) = u(k-1) + \Delta u(k) \tag{19}$$

$$\Delta u(k) = k_p xc(1) + k_i xc(2) + k_D \dot{xc}(1) \tag{20}$$

In order to adjust the PID controller parameters, RBFNN system architecture is shown in Fig. 5. Defined parameter index  $E(k) = \frac{1}{2} e^2(k)$ , and the value of formula is much smaller much better. By steep descent method, control parameters of PID can be derived by into the following formula (21).

$$\Delta k_p = -\eta \frac{\partial E}{\partial y_{out}} \frac{\partial y_o}{\partial \Delta} \tag{21}$$

Then, we can obtain (22).

$$\Delta k_p = \eta e(k) \tag{22}$$

where  $\frac{\partial y_o}{\partial \Delta}$  is the message of Jacobian, and we have

$$\frac{\partial y_{out}}{\partial \Delta u(k)} \approx \frac{\partial y_{out}}{\partial x(1)} = \sum_{j=1}^m w_j h_j \frac{c_j}{h_j} \tag{23}$$

RBFNN is single hidden layer with a three layer forward network [3]. He simulated the human brain partial adjustment, receptive field neural network. RBFNN can approximate any arbitrary precision continuous function. Use RBFNN to approximate nonlinear mathematical model of controlled body [1]. On the other hand, the flight controller on the quadcopter using PID control components. The attitude parameters include of vertical axis (Yaw), horizontal axis (Pitch), rolling (Roll), attitude stabilization (Alt). And sensor parameters have Balance (Level), Acceleration (Acc), Magnetometer (Mag), Throttle (Thro), and Gyroscopes (Gyro). The parameters of the rotating Pitch and Roll are most important. Outputs of Pitch and Roll will directly affect the output level and Alt, and it direct results in the outputs of Pos, PosR and NavR's[1][6].

### III. EXPERIMENTAL EVALUATION

In general, the linear mathematical model of the controlled body is difficult to fit varying flight conditions. The traditional PID controller cannot used to improve efficiently

the stable control problem of the quad copter. A control system with PID controller is presented in the following. As the quadcopter liftoff, acceleration output of Z axis measured by accelerometer with PID control as shown in Fig. 6, while the acceleration outputs of ROLL and PITCH measured by accelerometer and gyroscope as shown in Fig. 7, respectively. As the quadcopter taking off, the throttles of the motors can be varied to balance the attitude of the quadcopter. Note that all data of the experimental results are expressed within 0~10 seconds.



Fig. 8 Propeller driven by brushless DC motor being released to simulate the flight failure

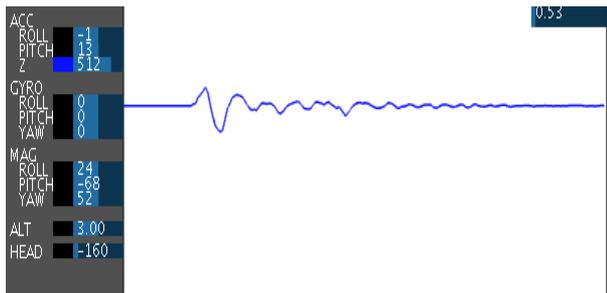


Fig.6 Acceleration output of Z axis measured by accelerometer with PID control.

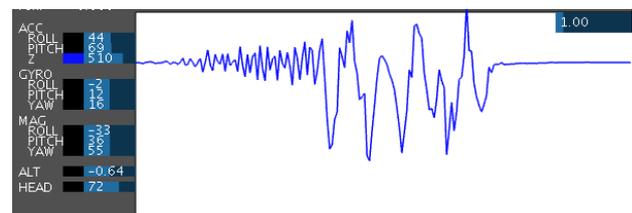
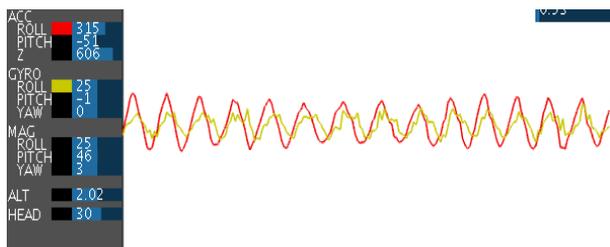
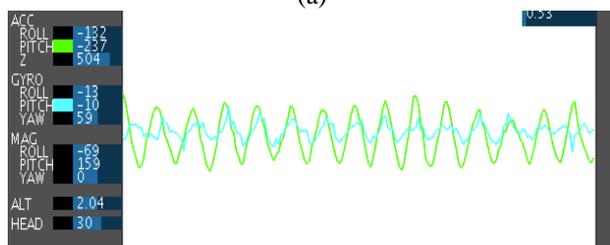


Fig.9 Acceleration response of the vertical axis with part failure under PID control

In Fig. 9, due to a propeller been released, the efficient power cannot be exported by the motor to cause idling, and the flight is out of control within 2.1 to 4.8 seconds. From 4.8 to 5 seconds of Fig. 9, it cause an extreme fall instantly, which results in the output of the barometer response, and more power is required to maintain the stable flight.



(a)



(b)

Fig.7 Control response of ROLL (a) and PITCH (b) when liftoff with PID control

As can be seen from Fig. 7, the flight status of the quadcopter is from transient into the steady state. Clearly, the vibration is difficult to be improved by PID control. The following demonstrates that the traditional PID control is applied to component failure during flight. In fact, acceleration of the vertical axis can be measured by the accelerometer. Observing the acceleration of the vertical axis from the accelerometer is based on the controller working. First, one propeller driven by brushless DC motor is released to simulate the flight failure as shown in Fig. 8. Then, let quadcopter liftoff slowly and the acceleration of the vertical axis is analyzed, where the measured result of the acceleration is shown in Fig.9.

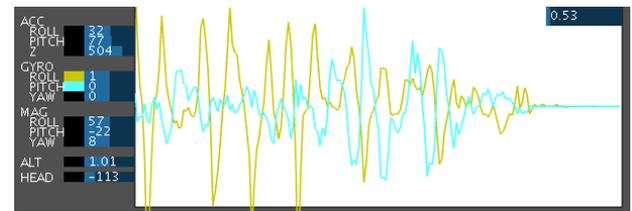


Fig.10 ROLL and PITCH responses measured by gyroscope with part failure under PID control

In Fig. 10, the flight is out of control within 2 to 7 seconds. The acceleration outputs of ROLL and PITCH measured by accelerometer and gyroscope. Because the released propeller is located on the forward side on ROLL axis. The flight attitude on ROLL axis is modified by the PID control system with failure. Due to inertia tilt, the attitude of the PITCH is regulated within 4 seconds. The throttle output of the motor can be much larger, which causes angular acceleration on Yaw axis rotation to exist. In the mathematical model of the controlled plant, the parameters of the controller are fixed. Therefore, in this paper, RBFNN based PID control is raised to adjust the parameters of PID controller.

#### IV. EXPERIMENTAL RESULTS

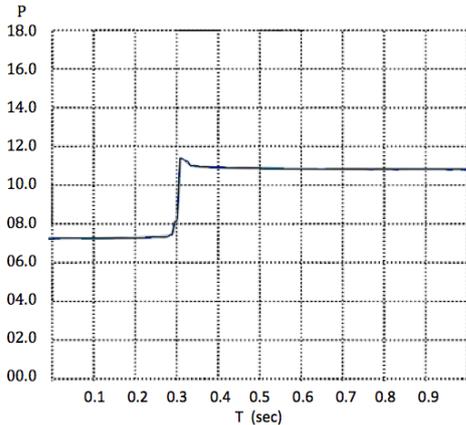
In this paper, RBFNN based PID control is proposed to adjust parameters of  $k_p$ ,  $k_d$ , and  $k_i$ . The regulation algorithm of the PID control is given as (23), (24) and (25).

$$k_p(k+1) = k_p(k) + \Delta k_p \quad (23)$$

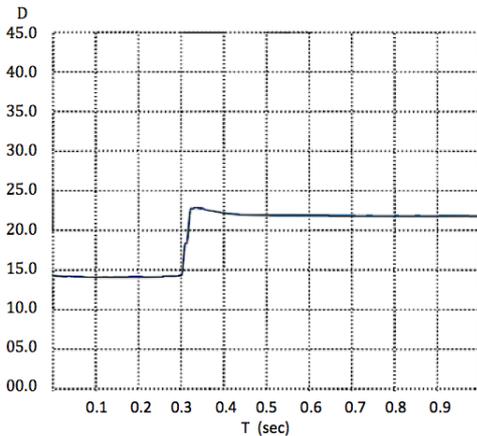
$$k_D(k+1) = k_D(k) + \Delta k_D \quad (24)$$

$$k_I(k+1) = k_I(k) + \Delta k_I \quad (25)$$

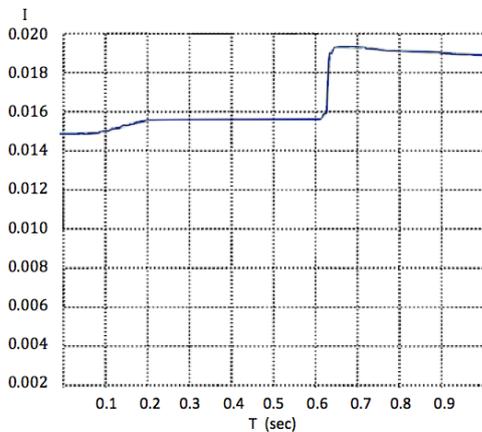
where  $\Delta k_p, \Delta k_D,$  and  $\Delta k_I$  are determined by RBFNN. As shown in Fig. 11, from 0 seconds to 0.3 seconds, the flight is normal, while starting at 0.3 seconds, the failure is given. The parameters of  $k_p, k_D,$  and  $k_I$  are modified by RBFNN.



(a)



(b)



(c)

Fig.11 Controller parameters of (a)  $k_p$ , (b)  $k_D$ , and (c)  $k_I$  tuning by RBFNN

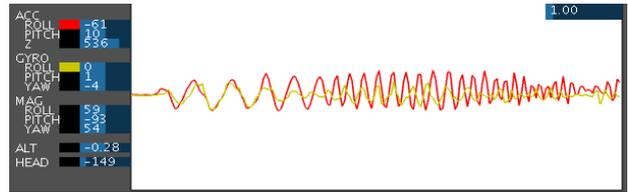
In Fig. 12, without any failure, adjusting the system parameters easy causes serious wrong flight status as quadcopter being liftoff. In 0 to 3 seconds of Fig. 12 after

liftoff, the parameters of the control system are not to zero, and the output of  $\Delta k_p, \Delta k_D,$  and  $\Delta k_I$  will be wrong. Therefore, the control system will not significant be affected by the acceleration output of the quadcopter.

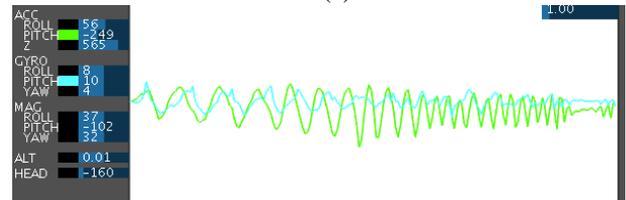


Fig.12 Acceleration response of vertical axis when liftoff with RBFNN-PID.

Compared to Fig. 7, the acceleration of the ROLL is less varied, but vibration frequency is increased as shown in Fig. 13. However, it can determine that quadcopter flight attitude is more stable but it still could not prove the RBFNN scheme to reach optimal value.



(a)



(b)

Fig.13 Acceleration responses of (a) ROLL and (b) PITCH when liftoff with RBFNN-PID

Although failure being difficult to be expected, RBFNN aims to be used to adjust the parameters of the controller to reach well diagnosis function. Experimental results of Fig. 14 show that the quadcopter can be still stable controlled under vibration. In fact, the control parameters of  $k_p, k_D,$  and  $k_I$  adjusted by RBFNN can be used to be the throttles of the motors, which can suppress the out-control state. In practical, the acceleration is decreased gradually after 5 seconds as shown in Fig. 14.

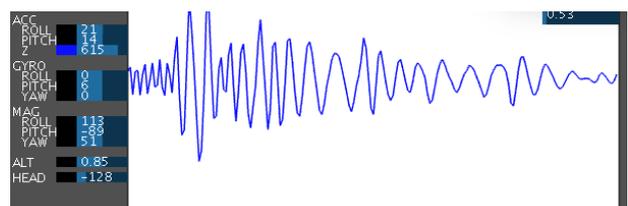


Fig.14 Acceleration response of vertical axis under component failure with RBFNN-PID.

On the other hand, observing accelerations of ROLL and PITCH as shown Fig. 15, the acceleration cannot be suppressed completely after out of control. The accelerations

of the ROLL and PITCH reach steady state after 5 time's abnormal acceleration.

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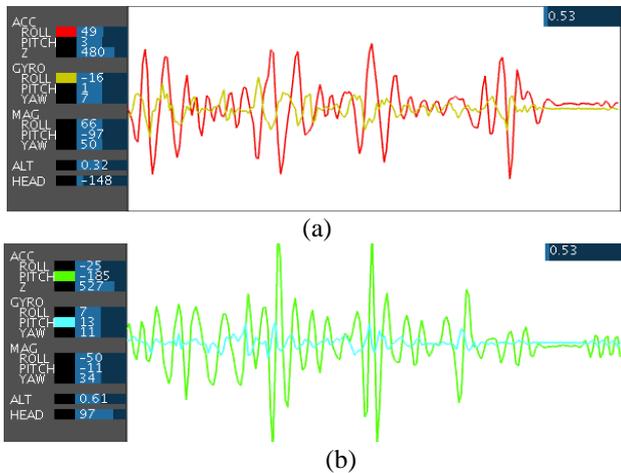


Fig.15 Acceleration response of (a) ROLL and (b) PITCH under component failure with RBFNN-PID.

### V. CONCLUSIONS

In this paper, a model of quadcopter dynamics has been driven and RBFNN based on PID controller was used to gradually decrease the vibration caused by out of control. From numerical simulation and experiment the proposed scheme was verified. PID control which include RBFNN control can suppress the vibration immediately when failure of structure. In experiment results, using RBFNN with PID control can suppress vibration by 5 time's abnormal acceleration. The proposed control scheme can be used to improve safety flying of the quadcopter. The proposed quadcopter control system can be modified to apply not only quadcopter but also hexacopter, even more rotor drone as multi-copter.

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